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Optimization of radiation protection in accelerators : decision making under uncertainty and risk

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Abstract : For medium and high energy accelerators cost optimization of shields and ventillation systems are to be considered from radiation protection point of view. Since, the optimum level of protection depends on the operating state (beam energy, beam current, extraction efficiency etc) of the accelerator, the best level of protection can be decided using the probability distribution of the operating states. The paper discusses the decision problem when no prior distribution is available (decision making under uncertainty), and when prior information is available about the distribution (decision making under risk).

Keywords : Radiation protection, accelerator, optimization, decision theory.

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1. Introduction

Optimization of radiation protection accelerators has gained importance from socio-economic point of view. Optimization is equivalent to carrying out some kind of balancing of the resources put into protection, and the level of protection achieved, within certain constraints, and taking into account certain factors—both radiological and non-radiological, including even political issues (ICRP 1989). As a result, optimization of radiation protection in nuclear installations becomes a complex decision making problem where the techniques and numerical analyses adopted are only decision aiding processes and the final recommendation is made only by the decision maker, using, of course, the quantitative data generated by those decision aiding techniques. In the present paper we consider, for the purpose of discussing the design of optimum level of protection, only those constraints and factors that are either radiological or related to the design and operation of the accelerator.

In medium and high energy accelerators, the radiation shield and ventillation systems are mainly to be considered for optimizing the level of protection. This is done in the cost-benefit approach by making a trade-off between the cost of protection and the cost assigned to the radiation induced detriment which is proportional to the radiation doses (ICRP 1983, Auxier and Dickson 1982). Radiation shields are necessary to protect workers from radiations (most importantly neutrons) generated due to accelerated ions striking various accelerator parts and targets. Ventillation systems are necessary to remove the radioactive ^{15}O and ^{13}N molecules that are produced near the accelerator by neutrons from $(n, 2n)$ reactions on ^{16}O and ^{15}N molecules present in air. It is evident, therefore, that the level of protection depends on the intensity and energy distribution of neutrons produced in the accelerator. The neutron flux intensity and neutron energy depend on the operating states of the accelerator (i.e. beam energy, beam current, extraction efficiency, type of accelerated ions etc). For designing optimal levels of protection, it is necessary to know the frequencies with which different operating states occur. That is, we should know that within a specified period the number of times the accelerator operates, for example, with 60 MeV alphas, 15% extraction efficiency, 1 μA required beam current, etc. The present paper discusses the use of entropy principles within the framework of cost-benefit approach to solve the decision problem that arises out of an incomplete knowledge of the frequency (probability) distribution of the operating states of an accelerator.

2. Cost benefit approach and related decision problems

The cost benefit approach is a decision aiding procedure which minimizes the total cost, $U(w, s)=X(w)+Y(w, s)$, where $X(w)$ is the cost of protection and $Y(w, s)$ is the cost assigned to the radiation induced detriment (Sarkar 1988, Sarkar and Muthukrishnan 1988). Here, w is the level of protection and s denotes the operating state of the accelerator. In fact, s represents several parameters related to accelerator operation, such as the extraction efficiency, the beam current, number of persons exposed to radiation and the neutron energy distribution. The neutron energy distribution depends on the energy and type of accelerated ions and the material on which the beam strikes. For a certain s the cost-benefit approach gives the optimum level of protection, w^*, as the level which minimizes $U(w, s)$. In accelerators where the parameters represented by s are variable, several states are possible and for each state there might be different optimum levels of protection. Let us assume that the number of possible states are n, and the state s_i $(i=1, \cdots n)$ occurs with probability p_i. We also consider that we have m different options of the level of protection from which we have to choose a particular one. In such cases, we can construct for finite n and m a two way table (Table 1) according to the classical decision problem.

The table indicates that if we select the level of protection w_j and the operating state happens to be s_i, then the total cost involved is $U(w_j, s_i)$. The best level of protection w^*, which minimizes the expected cost, is the solution of

$$\min_{\text{all } w}\left[\sum_{i=1}^{n} p_i U(w_i, s_i)\right] \tag{1}$$

which requires a knowledge of p, the probability distribution of s.

Table I. Two way table according to the classical decision problem.

Protection Options	$p_1\ p_2\cdots\cdots$ $s_1\ s_2\cdots\cdots$	p_i s_i	$\cdots p_n$ ←Probabilities $\cdots s_n$ ←Operating states
w_1 w_2 ⋮			
w_j		$U(w_j, s_i)$	
⋮ w_m			

If the decision maker does not know a prior probability distribution, then this decision problem is known as decision making under uncertainty. The Laplace criterion based on the principle of insufficient reason can be used to solve such problems. The principle says that all states are equally likely, i.e., $p_i = 1/n$, for all i, if we donot have sufficient information to conclude otherwise. When a prior probability distribution is known to the decision maker, the decision problem is classified as decision making under risk. Entropy principles can be used with advantage to solve both types decision problems (Buckley 1985). We now illustrate the use of maximum entropy and minimum cross entropy principles in decision problems connected to the optimization of radiation protection in accelerators.

3. The entropy principles

We assume that the status $s_1, s_2 \cdots s_n$ of the system are mutually exclusive and exhaustive and prob $(s_i) = p_i$, for all i such that $\sum_{i=1}^{n} p_i = 1$ and $p_i \geqslant 0$. A measure of the uncertainty in the decision problem is computed by $H(p) = H(p_1, p_2 \cdots p_n)$ where

$$H(p) = -\sum_{i=1}^{n} p_i \ln (p_i) \tag{2}$$

We define, $0 \ln (0) = 0$. $H(p)$ is also known as the entropy of the decision problem (Buckley 1985).

The maximum entropy principle states that the decision maker should use the probability vector, p^*, that has the maximum entropy, i.e., p^* solves

$$\max_{\text{all } p} \left[-\sum_{i=1}^{n} p_i \ln (p_i) \right] \tag{3}$$

When the decision maker does not have any knowledge about the probability distribution, then (3) is solved without any constraint and the solution is $p_i^* = 1/n$, for all i, which agrees with the Laplace criterion.

In the present case, the decision maker may not be completely ignorant of the possibilities and may have some prior information about p_i without knowing their exact values. The prior knowledge is likely to be in the form of interval estimates i.e., $0 \leqslant a_i \leqslant p_i \leqslant b_i \leqslant 1$ for all i. For example, the probability of 20% extraction efficiency for 60 MeV alphas may be in the interval 0.1 to 0.3. The decision maker might also get an interval estimate from experts or experienced persons. In some situations, the decision maker might obtain some prior probability distribution q from an existing accelerator that need not be exactly similar. This prior q, along with some available information (used as constraints), will give the posterior distribution p from the entropy principles. In such cases a quantity $H(p, q)$, called the cross-entropy (Good 1963) is defined as follows :

$$H(p, q) = \sum_{i=1}^{n} p_i \ln (P_i/q_i), \tag{4}$$

where it is assumed that $q_i > 0$ for all i, and $H(p, q) = 0$ if $p = q$, otherwise it is positive. The minimum cross-entropy principle states that the decision maker should now use the posterior probability, p^*, that minimizes $H(p, q)$, subject to satisfying new constraints. It may be noted here that if q is uniform (i.e. $q_i = 1/n$, for all i) then the minimum cross-entropy principle reduces to maximum entropy principle.

The entropy principles can be adopted in deciding optimum protection levels when we have limited information about the future operating states of the accelerator. The method is advantageous because exact probability distributions are not needed to start with. The principles generate unique priors from available information and thus translates decision making under uncertainty to decision making under risk. A detailed study of the use of entropy principles in decision problem has been made by Buckely (1985).

4. Conclusion

Optimization of radiation protection in accelerators finally reduces to a decision making problem. The cost-benefit approach along with the entropy principles can be adopted to arrive at some solutions.

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